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A MODEL FOR INFORMATION EXCHANGE

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ABSTRACT: Information exchange theory aims at the study and modeling of information exchange processes among interacting agents. In this paper, we develop a model for information exchange. The concepts of protocols, types of information systems, misinformation and information distortion, codification, and information distance are introduced. Examples of information exchange processes are given. In the appendix, a model for competitive information exchange is presented. The estimation of alien models for this competitive information exchange model is described.

KEYWORDS: Information exchange theory, communications theory, theory of protocols, conversational models, knowledge-based systems.

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1. Introduction

Information exchange theory aims at the study and modeling of information exchange processes among interacting agents. An agent can acquire knowledge about the outside world in two ways: (a) engagement in direct action, and (b) exchange of information with other agents.

Engagement in direct action is the primary means of acquiring knowledge, whereby experiences are acquired and accumulated. Experiences, as primary source of knowledge, are distilled and abstracted to construct a knowledge base — understanding of the outside world. This knowledge acquisition process is feasible only for those agents capable of engagement in direct action.

However, not all agents are capable of knowledge acquisition via direct action. The second knowledge acquisition process is through exchange of information with other agents.

As illustrated in Figure 1, in the knowledge acquisition process, experiences are abstracted to construct a knowledge base. This abstracted information representation in the knowledge base has several advantages over raw experiences: (a) economy of representation; (b) expressiveness; and (c) efficiency in knowledge transmission. By communicating with other agents using this abstracted information representation, other agents do not have to rely upon direct action in order to acquire knowledge.

The process of information exchange can be seen to be an indirect (and often more efficient) way of constructing a knowledge base about the outside world. As illustrated in Figure 2, each agent has its own knowledge base and experiences pool. Information exchange can be seen

to be a replacement and/or supplement to direct action in the real world.

In this paper, we develop a model for information exchange. In Section 2, the basic concepts of information exchange theory are introduced. Protocols, types of information systems, misinformation and information distortion, codification process, and information distance, are discussed in Sections 3, 4, 5, 6 and 7, respectively. Examples of information exchange processes are given in Section 8.

The appendix describes a model for competitive information exchange. The estimation of alien-models for this competitive information exchange model is described in Section A2 and A3. Section A4 introduces more complicated models. Finally, in Section A5, neagent message exchange is discussed.

This paper is the first part of three papers on information exchange theory. The second part discusses the analysis and synthesis of information exchange protocols. The third part deals with the application of information exchange theory to the design of message filters, and the specification and decomposition of user models.

2. Basic Concepts

The basic concepts of information exchange theory will now be introduced to characterize the information exchange process. A formal model is presented at the end of this section.

We define an <u>agent</u> as some active process (a person, a computer program, an organization, etc.), which is capable of manipulating <u>data</u> and assuming different <u>states</u>. In the above, <u>agent</u>, <u>data</u>, and <u>state</u> are primitive concepts. We define <u>information</u> as data which, when manipulated by an agent, enables the agent to assume or change states.

The process of information exchange between two agents is called a conversation or interaction. The unit of information exchange is called a message. The basic dyadic conversational model is illustrated in Figure 3 (Note 1). In Figure 3, each agent has access to a data base. Each agent also has a model of itself, called the self-model, and a model of the other agent, called the alien-model. The model of the alien agent may range from being very precise to rather vague (closely coupled to loosely coupled). The concept of an alien-model in information exchange theory is important. This alien-model enables one agent to couple its information gathering and processing activities with another agent's corresponding activities. The coupling process is called association, which enables the two communicating agents to share their experiences and achieve synergy (Note 2).

If the (self-model, alien-model) pair of two communicating agents become reciprocally identical after they have correctly estimated the respective alien-models, then the dyadic system is said to have achieved synergy or resonance.

The function of the encoder is to encode messages for transmission via a communication channel, and that of the decoder is to decode the transmitted messages. Sometimes the encoder is used to transmit misinformation, and the decoder is then used to detect the transmission of misinformation. The usage of misinformation will be discussed in Section 5.

An <u>information node</u> consists of an agent (with its data base, self-model, alien-model, encoder and decoder) and communication channels to other information nodes. An <u>information</u> system is a network of interconnected information nodes allowing information exchange among interacting agents.

We now present a formal definition of an agent as follows.

<u>DEFINITION 1</u>: An agent M is a 9--tuple (X,S,g,h, so, F, E, D, DB), where

X is a nonempty set of messages (the message space)

S is a nonempty set of states (the state space), S = Sr x Ys x Ya, and Sr is a nonempty set of true states of agent

Ys is a nonempty set of self-models

Ya is a nonempty set of alien-models

g: Sr x (X x X)* -> Sr is the true state transition function
 (given current true state and history of input/output message
 exchanges, g specifies the next true state)

h: S x (X x X)* -> S is the state transition function (given current state and history of input/output message exchanges, h specifies the next state)

so in S is the initial state

F is a subset of S, the final states

E: S x X -> X is the encoder which maps true output message to external output message

D: S x X -> X is the decoder which maps external input message to true input message

D3 is a data base which stores history of input/output message exchange, DB is a subset of $(X \times X)^*$

As a convention, the message space X contains a <u>null message</u>, e, so that an agent can send out messages spontaneously, or receive

messages without having to send an immediate response. Thus (xi, e) indicates M receives a message xi, (e, xj) indicates M sends out a message xj, and (xi, xj) indicates M sends out a message xj after having received message xi.

The true state transition function g can be derived from the state transition function h as follows, I(3,1)(h((wi,wj),s,ys,ya)) =g((wi,wj),s) for any (wi,wj) in $(X \times X)*$, s in Sr, ys in Ys, and ya in Ya, where I(3,1) denotes the projection of a 3-dimensional space onto its first coordinate. This also implies that h must be consistent, in the sense that for any уs in Ϋ́s and in ya Ya, I(3,1)(h((wi,wj),s,ys,ya)) are identical.

Let w1, w2 be arbitrary message strings in $(X \times X)^*$. If h(so,w1) = s1 and h(so,w1w2) = s2 implies h(s1,w2) = s2 for any w1 and w2, then the state transition function h can be defined as a function from $S \times X \times X$ into S, i.e., the next state depends only on the current state and the immediate message exchange.

The set of self-models, Ys, can be defined as the power set of Sr

Sr, or 2 . Similarly, the set of alien-models, Ya, can be defined as Sr'

the power set of Sr', or 2 , where Sr' is the true state space of other agent. In other words, if M is not sure about its true state, ys represents the set of states which it thinks contains its true state. In general, Ys and Ya can be arbitrary models of M.

DEFINITION 2: Given two agents Mi and Mj, let (xil, ..., xin) denote the sequence of messages from Mi to Mj, and (xjl, ..., xjn) the sequence of messages from Mj to Mi. A conversation or interaction is the combined sequence of messages ((xil,xjl), ..., (xin,xjn)). A

protocol is the set of rules governing a conversation, during which Mi and Mj attempt to estimate self-model and alien model. If after protocol exchange, Mi is in state (si,{si},(sj}) and Mj is in state (sj,{sj},{si}), then the two agents have achieved synergy or resonance.

3. Protocols

A set of rules governing a conversation to enable interacting agents to establish respective models of alien agents and to form association is called a protocol. It should be noted that association refers to the coupling of models, and protocol is the set of rules governing message exchanges to form this association. Protocols are very important in conversation. Quite often, protocol exchanges are the only type of conversation sustained. Protocols can be used to maintain alien-models. Social relationships such as friendship often need the reinforcement by protocols or ritualistic behavior.

Protocols often can be subdivided into many levels. At each level, a different pair of self-model and alien-model is assumed. Lower-level protocols become building blocks for higher-level protocols. As an example, social protocols are built upon greeting protocols. As another example, in a computer-communication network, one can identify and define many levels of protocols, ranging from physical level, link control level, network control level, to user level protocols [BACHM78].

Protocols also change in time, with corresponding changes in alien-models. For example, when two strangers first meet, they use a polite greeting protocol. When they become close friends, the greeting protocol changes accordingly. When they become lovers and finally husband and wife, greeting protocol changes again.

Different modes of interaction can be studied, by studying the following: (a) how the alien-models are estimated, i.e., the process of mo'el associa ion; (b) how the alien-models change in time; and (c)

efficacy and efficiency in information exchange, i.e., the efficacy and efficiency of obtaining the information solicited in an information exchange process.

Agents can be further classified into several types: (a) Collector, or information sink, which solicits and collects information and stores information in its data base; (b) Provider, or information source, which provides information solicited by other agents; (c) Analyzer, which processes data stored in its data base to generate output information; (d) Filter, which condenses and compresses input information to generate output information; (e) Annihilator, which destroys input information; (f) Creator, which spontaneously creates information. Agents may assume different roles at different occasions. For example, a public relation person is an Annihilator; a secretary is a Filter in one role, and a Collector in another role; a library system serves as a Provider; a company executive is an Analyzer; and an artist is a Creator.

Protocols among agents can be classified based upon their respective types into P->C, A->C, F->C, P->A, P->F, P->P, A \leftarrow >A, A->F, and F->F protocols.

Given a protocol exchange (xi1, xj1; xi2, xj2; ...; xin, xjn), we can investigate how this protocol exchange can be used to estimate the alien-models of interacting agents. It is here assumed that (a) agents do not change state during protocol exchange, and (b) encoder E and decoder D perform identity mappings.

For any agent M, two true-states s1 and s2 are said to be equivalent with respect to a message pair (iii, ji), if

g(sl,(xik,xjk)) = g(s2,(xik,xjk)). This equivalence relation is reflexive, transitive, and symmetric. Let A(xik,xjk) denote the set of equivalent true-states of an agent with respect to a message pair (xik,xjk). Let A denote the intersection of A(xil,xjl), ..., A(xin,xjn), or

A = (xik,xjk) is
a message pair
in a protocol
exchange

If A is a singleton set {sn}, then sm is the true state of an agent during protocol exchange. If A contains more than one state, then the protocol exchange can only be used to estimate the class of an agent. After protocol exchange, the agent is identified to be a "class A agent". For the other agent, its alien-model ya can be estimated as A.

The situation becomes much more complicated, if agents change states during protocol exchange, or encoders are used to distort information. Such topics are of interest for further research.

It should be noted that we do not necessarily need to estimate the true state of an agent. Often we only need to know what class of agent it is, i.e. the equivalent states, and information exchange can begin. A person with great "telephone personality" can often obtain useful information over the telephone. The efficacy and efficiency of information exchange by telephone is also due to the fact that simple protocols are used in telephone conversation, to construct "crude" alien-models. On the other hand, only certain information can be exchanged that way. Further information exchange requires better model associations, i.e., more precise alien-models.

4. Classification of Information Systems

Information systems can be classified, according to the number of interacting agents, into three categories: (a) Dyadic systems, with two interacting agents. The dyadic system is the most basic model of information exchange. (b) Small group systems, with three to nine interacting agents. (c) Large group systems, with more than two teracting agents.

In f rmation systems can also be classified, according to the control structure, into three categories: (a) Tutorial system: One agent has control and provides information to other agent or agents. Other agent(s) may ask questions to obtain answers from the tutoring agent. The tutoring agent may also provide tests and ask questions to find out whether the other agent(s) has obtained the information. (b) Interview system: One agent has control and asks questions to ascertain the state(s) of other agent(s). (c) Message interchange system: Participating agents share control and linearct with each other to exchange information. When interacting agents have no way to predetermine the areas of expertise and ignorance of sech other, shared—control message exchange becomes important.

Each agent in an information system may be: (a) a person, (b) a machine, (c) a codified agent (see Section 6), or (d) a complex organization.

The above classification schemes provide a matrix of reference to classify information systems.

(1) Dyadic system can be tutorial dyadic system, interview dyadic system, or interchange dyadic system.

- (2) Small group system can be panel discussion group (tutorial), Delphi group (interview), or brain-storming group (interchange).
- (3) Large group system can be public lecture group (tutorial, using rhetoric techniques) or participatory democracy group (interview). Large group system is essentially a one-way communicating system, with limited feedback using polling and rating techniques. A "fire-side chat", for example, is a large system tutorial camouflaged as a dyadic interchange.

Current man-machine information systems are either dyadic tutorial systems (such as computer-aided instruction systems) or dyadic interview systems (such as information retrieval systems). This is because for these two types of systems, the control structure is simple (one agent has control), and the alien-model can be predefined. Take the Eliza system as an example. This is a dyadic interview system, with Eliza as the interviewer, and the human participant as the interviewee. Since the alien-model for Eliza has been taken for granted (i.e. predefined) by the human participant, Eliza passes as a psychiatrist. The purpose or goal for Eliza is also taken for granted. The conversation created by Eliza is not a true conversation, but a pseudo-conversation.

5. Misinformation and Information Distortion

Misinformation can be defined as the deliberate distortion of information by an agent (Note 3). In a large system for one-way communication, information distortion is inevitable, because the system must distort by selection. On the other hand, the receiving agent can also distort a message because of its information bias.

There are two types of <u>information bias</u>: (a) bias of self-model, and (b) bias of alien-model. A political leader may be regarded as a saint by his own countrymen (bias of self-model), and a villain by foreigners (bias of alien-model). Protocols can sometimes be used to negate information bias of the underlying self-model or alien-model.

In many situations for information exchange, the misrepresentation of information, or the sending of misinformation, is also very important. Misinformation can be exchanged (a) in bargaining protocols, (b) as a tool in policy control, and (c) to avoid undesirable consequences.

In bargaining protocols, to achieve some desirable goals or objectives, agents may send false information, or tell lies. For example, in "shopping protocol", the customer pretends not to buy, so as to induce the seller to lower the price. The seller also pretends not to sell, so as to induce the buyer to raise the offering price. Misinformation is exchanged, in order to estimate the true alienmodels. Similarly, in international politics, foreign policy is often meant to change other agent's model of oneself. However, inaccurate alien-models may lead to misinterpretation of messages from alien agents [SOLOM71].

In policy control, the government can send carefully prepared forecast about future outlook of economy to the public (households, firms and banks, etc.) to induce the public to react in a certain way, thereby achieving economic stability [TAM79]. For example, the Carter administration proposed tax rebate on January 29, 1977. When the public reacted favorably, resulting in less unemployment, the Carter administration withdrew the tax rebate proposal on April 14, 1977. This is another example of the usefulness of misinformation. Similarly, in price control, government's inflation forecast will influence the workers' union to change its price expectations and wage demands.

Misinformation is also often used to avoid undesirable consequences. In a success-oriented society, failures cannot be tolerated without losing credibility. Misinformation is needed to induce resource expenditures by other agents, to maintain an alien-model having a "success image". Project and product promotion often works this way. The Edsel syndrome and the Pinto syndrome are good examples. Vietnam War is yet another example of using misinformation to induce further resource expenditures to transform "failure" to "success". On a smaller scale, a professor may also use misinformation to induce a student to continue Ph.D. research work, by telling him he is very near to completion — which may not be the truth.

In the model described in Section 2, the encoder and decoder processors form the conversion of true messages to external messages containing misinformation, and conversely.

6. Codification Process

In an information system, not all interacting agents are active agents. Some may be codified agents, such as books and motion pictures.

Books can be regarded as information nodes in an information system, with author as agent, author's model of reader as alien-model, and author's knowledge as his data base (Note 4). The reader is the alien agent communicating with the book as a codified agent. The codification process (a) delimits and structures data, (b) determines model of alien agent, and (c) determines mode of communication. The mode of communication in this example, is the reading of the book by the reader.

Similarly, a motion picture can also be regarded as a codified agent, with director as agent, director's conception of audience as his alien-model, and director's knowledge as his data base. The audience becomes the alien agent, and the mode of communication is the watching of the motion picture by the audience.

Therefore, the codification process converts a dyadic system to a large group system with one-way communication. The codified dyadic system differs from a large system in two respects: (a) A large system generally operates by message broadcasting. It is a time-synchronous system. The codified dyadic system is a diachronic system (a book can be read many times, by past, present or future readers). (b) A large system may allow some feedback and message interchange, whereas the codified dyadic system does not allow any feedback.

An information system may also become codified for an external

observer. This codified system conveys new (and often unintended) messages to the observer. Again, the characteristic of the codified system is that only one-way communication is allowed, i.e. messages are transmitted to the observer, but not conversely.

7. Information Distance

In an information system, some interacting agents exchange information more often among themselves, although they may be geographically, structurally distant from one another. Examples of such subgroups include: sub-cultures, professional societies, cliques and peer groups (Note 5). In addition to a measure of <u>rate</u> of information transmission, there is a need to define a measure of information distance.

Let Mi and Mj be two communicating agents, Pij the probability of Mi sending a message to Mj, and Pji the probability of Mj sending a message to Mi. The <u>skewed information distance</u> from Mi to Mj, ds(Mi,Mj), is defined to be 1/Pij, and the skewed information distance from Mj to Mi, ds(Mj,Mi), is defined to be 1/Pji. The information distance between Mi and Mj is defined to be d(Mi,Mj) = 1/(Pij x Pji), or the product of the skewed information distances ds(Mi,Mj) and ds(Mj,Mi). The smallest information distance d(Mi, Mj) is 1. When either Pij or Pji is 0, d(Mi, Mj) becomes infinity.

In Section A5 of the appendix, the formation of communicating sub-groups that reflect the concept of information distance will be discussed. A preliminary definition of information distance between two information nodes is also presented in that section.

8. Examples of Information Exchange Processes

In this section, we draw examples from various disciplines to illustrate the information exchange processes.

In social psychology, dyadic social interaction can be modeled as incremental information exchange between two agents [HUESM76]. Each agent has a number of psychological states, which is assumed to be linearly ordered from "shallow states" to "deep states". Message spaces and payoff functions vary from state to state. An action protocol determines state transitions — message exchanges that are mutually beneficial can lead to transitions to deeper states, resulting in deeper psychological involvement and more intimate relationships. This model can be used to simulate various behavior patterns in social psychology.

In transactional analysis [BERNE73], a transaction is defined to be a unit of dyadic social interaction of message exchange between two agents. The interaction is determined by interpersonal psychological state pairs. Each agent has a goal state or final state. He conceives a sequence of state changes and message exchanges to lead to some desirable goal state. The messages are then encoded to become surface message exchanges. In transactional analysis, the aim of the psychologist is to reverse the process, i.e. to decode surface message exchanges to understand true message exchanges, and the true state pairs in message exchange. For example, a surface message exchange may be interpreted as the interaction of one agent in "parent-state" with another agent in "child-state".

In economic theory, we have economic agents exchanging messages

which are either price information or resource exchange information [HAYEK45]. The economic environment defines the characteristics of an agent: its preference relation, initial endowment, and technology set. The adjustment process then specifies the message space for agents, and the response rules (the adjustment mechanism) that the agent must follow [HURWI59, CAMAC70]. One can then study problems such as economic equilibrium, informational efficiency, and system performance characteristics of a given economic environment satisfying certain properties [TINBE67]. One can also study the optimal prestructuring of information exclude protocols for decision making [ALBIN81].

In the above, we have given examples of information exchange involving two or more active agents. The interacting agents may engage in team work toward some common objective [MARSC72, GROVE73], or may be competitive and antagonistic to one another, as will be discussed in the appendix.

9. Discussion

This paper has presented fundamental concepts of information exchange theory. It is hoped that the conceptual framework presented in this paper may lead to systematic ways of understanding the information exchange processes in complex organizations or social systems. As an example, our understanding of a complex organization usually combines several types of information exchange processes: (a) system-level messages in the form of measurements of system state indicators (number of employees of an organization, average salary, age and race distribution, etc.); (b) dyadic message interchanges (interview of individual members from an organization); (c) group interviews (polling and rating); (d) messages from collified agents or codified information systems (books, reports, statement from employees' union, etc.); and (e) tutorials (news broadcast, television news, etc.) (Note 6).

Information exchange process has two other interesting abstractions: (a) Information exchange usually happens at several levels simultaneously. Message exchange at one level is usually interpreted and acquires significance and meaning at a different level. This multi-leveledness is apparent in many information exchange processes.

(b) Information exchange usually is multi-directional, with intentional effects as well as unintentional side effects. Information transmission is often spontaneous — although the interacting agents may not intend to exchange information in a certain way, it happens often to be the case. This again is due to the multi-level nature of the information exchange process. It is also related to the codification process discussed in Section 6. As a result, The efficacy and

efficiency of an information exchange process can be evaluated in multiple ways, depending on the level of message exchange of interest, and the direction of message exchange of interest.

In conclusion, the study of how different types of information exchange processes are combined, how such messages are stored in a knowledge base, and how to perform multi-level, multi-directional evaluation of the efficacy and efficiency of an information exchange process, will be of interest for further investigation.

Notes:

- 1. An information exchange system may utilize a telecommunication system, or a transportation system, or a combination of both, to make message exchange feasible. Computer information systems are also utilized to facilitate information exchange, or to implement an information exchange system.
- 2. The Latin root for communication, communico, means "to share".
- 3. "The lie is the specific evil which man has introduced into nature", Martin Buber, Good and Evil, Scribner's, 1973, pp. 7.
- 4. "These are not books, lumps of lifeless paper, but minds alive on the shelves. From each of them goes out its own voice, as inaudible as the streams of sound conveyed day and night by electric waves beyond the range of our physical hearing; and just as the touch of a button on our set will fill the room with music, so by taking down one of these volumes and opening it, one can call into range the far distant voice in time and space, and hear it speaking to us, mind to mind, heart to heart." Gilbert Highet
- 5. World-wide information system need not lead to universal conformity. It may also lead to deliberate nonconformity.
- 6. In dynamic social systems, information exchange processes also exhibit time-dependent changes. For example, during the initial phase of a political movement, there usually is a sudden increase in the need to communicate among participants (publication of pamphlets, meetings, public lectures, etc.). When synergy is achieved and the participants share similar self-models, alien-models and goals, the need to communicate decreases. After the movement is institutionalized, the usual information exchange processes are reestablished.

References:

[ALBIN81] P. S. Albin, "Optimal Prestructuring of Information Exchange in Multi-Criterion Decision Making", Proceedings of 1981 International Symposium on Policy Analysis and Information Systems, Taipei, Taiwan, August 20-22, 1981.

[BACHM78] C. W. Bachman, "Provisional Model for Open System Architecture", Proc. of 3rd Berkeley Workshop on Distributed Data Management and Computer Networks, August 29-31, 1978.

[BERNE73] E. Berne, Games People Play, Ballantine Books, New York, 1973.

[BOULD56] K. Boulding, The Image: Knowledge in Life and Society, Univ. of Michigan Press: An Arbor, 1956.

[CAMAC70] A. Camacho, "Externalities, Optimality and Informationally Decentralized Resource Allocation Processes", International Economic Review, Vol. 11, No. 2, June 1970, 318-327.

[CHANG79] S. K. Chang, "Toward a Theory of Information Exchange", Proceedings of First International Symposium on Policy Analysis and Information Systems, June 28-30, 1979, 252-264.

[GROVE73] T. Groves, "Incentive in Teams", Econometrica, Vol. 41, No. 4, July 1973.

[HAYEK45] F. Hayek, "The Use of Knowledge in Society", American Economic Review, Vol. XXXV, No. 4, September 1945, 519-530.

[HUESM76] L. R. Huesman and G. Levinger, "Incremental Exchange Theory: A Formal Model for Progression in Dyadic Social Interaction", Advances in Experimental Social Psychology, Vol. 9, Academic Press, 1976, 191-229.

[HURWI59] L. Hurwicz, "Optimality and Informational Efficiency in Resource Allocation Processes", in K. J. Arrow et al. eds., <u>Mathematical Methods in the Social Sciences</u>, Stanford University Press, 1959, 27-46.

[MARSC72] J. Marschak and R. Redner, The Economic Theory of Teams, New Haven Yale University Press, 1972.

[SOLOM71] R. Soloma, Mao's Revolution and the Chinese Political Culture, University of California Press, 1971.

[TAM79] M. Tam, "Public Forecasts: A Tool of Economic Control", Proceedings of First International Journal on Policy Analysis and Information Systems, August 1979.

[TINBE67] J. Tinbergen, Economic Policy: Principles and Design, North-Holland Pub. Co., Amsterdam, 1967.

[YAGER79] R. R. Yager, "On Modeling Interpersonal Communications", in <u>Fuzzy Sets: Theory and Applications to Policy Analysis and Information Systems</u>, S. K. Chang and P. P. Wang (eds.), Plenum Publishing Co., 1979.

Appendix: Competitive Information Exchange

Al. Competitive Information Exchange Model

In what follows, we will describe a model for competitive information exchange. Competitive information exchange situations arise in economic message exchanges and in game theory. In our model, each agent Mi has a payoff function associated with exchanged messages as follows:

where ai ar -1, 0 or +1, and a0+a1+a2+a3=0. In dyadic competitive message exchange, each agent will take turns in sending messages to the other agent, who then responds. In other words, agent '! sends xi to agent Mj, who responds by sending back xj. The payoff to Mi is obtained from the payoff function fi. Conversely, Mj may also send xj to Mi, who responds by sending back xj. The agent with the highest accumulated payoff is the winner.

As an example, agent M1 has the following payoff function:

Similarly, agent M2 has the following payoff function:

	× 2	×1	payoff
	0	0	1
f 2	0	1	-1
	i	n	1
	1	1	-1

If M1 sends 0 to M2, who responds by sending back 0, then the payoff to M1 is 0 (using payoff function f1), and the payoff to M2 is 1 (using payoff function f2). In other words, each agent uses his own payoff function to calculate his payoff.

In general, each agent Mi knows only his payoff function fi —

i.e. he has an accurate self-model. Therefore, each agent may attempt
to optimize his payoff by selecting appropriate action rules with
respect to his payoff function. A reasonable strategy is to send and
respond to messages by optimizing expected payoff.

DEFINITION 2: The optimal expected payoff rules for Mi are:

(Mi·1) send 0 if a0+a1 > a2+a3 (expected payoff E1 is 0.5(a0+a1))

send 1 if a0+a1 < a2+a3 (expected payoff E1 is 0.5(a2+a3))

send 0 or 1 if a0+a1 = a2+a3 (expected payoff E1 is 0.5(a0+a1))

(Mi·2) when receive 0, respond 0 if a0 > a2 (expected payoff E2 is a0)

respond 1 if a0 < a2 (expected payoff E2 is a2)

respond 0 or 1 if a0 = a2 (expected payoff E2 is a0)

(Mi.3) when receive 1, respond 0 if al > a3 (expected payoff E3 is al) respond 1 if al < a3 (expected payoff E3 is a3) respond 0 or 1 if al = a3 (expected payoff E3 is al)

The overall expected payoff E is given by the expression:

(1) E = 0.5E1 + 0.25E2 + 0.25E3= $0.5 \max(a0+a1,a2+a3) + 0.25 \max(a0,a2) + 0.25 \max(a1,a3)$

For example, MI may select the following action rules:

- (M.1) send 0 (expected payoff E1=0.5)
- (M1.2) when receive 0, respond 0 (expected payoff E2=0)
- (111.3) when receive 1, respond 0 (expected payoff [3=1)

and the overall expected payoff E = 0.5. On the other hand, M2 may select the following action rules:

- ($1.2 \cdot 1$) send 0 or 1 (expected payoff E1=0)
- (M2.2) when receive 0, respond 0 or 1 (expected payoff F2=1)
- (112.3) when receive 1, respond 0 or 1 (expected payoff 1 = -1)

and the overall expected payoff E = 0. Such action rules will guarantee that optimal expected accumulated payoff is obtained if the opponent sends and responds to messages randomly.

DEFINITION 3: An agent who behaves according to the optimal expected payoff rules is called an <u>Honest George Agent</u> (HGA).

It should be noted that the state space S of an HGA consists of a single element (fi,fi,*), where fi is its self-model, and * is a special symbol denoting a random opponent.

THEOREM 1: The expected overall payoff E of an honest george agent
.
(HGA) facing a random opponent is always nonnegative.

PROOF: There are only 19 different payoff functions (1 with all 0's, 12 with one 1, and 6 with two 1's). We can enumerate these payoff functions to prove that the expected overall payoff F (equ. (1)) of NCA is always nonnegative. The nineteen payoff functions are listed in Table Al:

function no.	aθ	a l	a 2	a 3
0	1	– i	ı	-1
1	-1	1	-1	1
2	n	0	n	0
3	1	-1	0	0
4	-1	1	0	0
5	0	0	1	– I
6	0	0	-1	1
7	i	~ l	-1	1
8	-1	1	1	-1
9	1	O	0	-1
10	- i	С	0	1
11	()	1	– 1	0
12	n	-1	1	0
13	1	0	-1	0
14	-1	0	1	0
15	0	-1	0	1
1.6	n	1	0	-1
17	1	1	-1	~1
18	- i	-1	1	1

Table Al Nineteen payoff functions

Alternatively, we can prove the above theorem analytically as follows. First, we observe that E1 > 0. Suppose not, then from equ. (1), there are two cases: (i) E1 = a0 + a1 < 0. Since a0 + a1 + a2 + a3 = 0, we have a0 + a1 = -(a2 + a3) < 0, or a2 + a3 > 0 > a0 + a1, implying that E1 = a2 + a3 > 0, a contradiction. (ii) E1 = a2 + a3 < 0. Similar argument leads to a contradiction. Next, suppose that E < 0. From equ. (1), we have 0.5E1 + 0.25E2 + 0.25E3 < 0. Since E1 > 0, we must have E2+E3 < 0. There are two cases: (i) E2 < 0. From equ. (1), a0=a2=-1. Since a0+a1+a2+a3=0, we must have a1=a3=1, and thus E3=1. But then, E2+E3=0, a contradiction. (ii) E3 < 0. Similar argument leads to a contradiction. Therefore, E cannot be negative. 0.E.D.

The above theorem says that the expected payoff for an HGA against any opponent is always nonnegative, (Honest George is basically an optiministic fellow!) and the optimal expected payoff is actually attained, if the opponent is a random agent (which is equivalent to an HGA with payoff ai's identically zero).

When both M1 and M2 in the above example are honest Coornes, then their message exchanges are determined by the optimal expected payoff rules. For the above example, the exchanges are: (a) M1 sends 0, and M2 responds with 0 or 1; (b) M2 sends 0 or 1, and M1 responds with 0. Therefore, their respective payoffs can be tabulated as follows:

```
M1-send=0
            "2-response=0
                             111-payoff = 0
                                              112-payoff= 1
                             M1-payoff = 1
                                              112-payoff = 1
1!1-send=0
            M2-response=1
112-send=0
                             M1-payoff=0
                                              M2-payoff = 1
            M1-response=0
M2-send=1
            M1-response=0
                             M1-payoff= 1
                                              1.2-payoff= 1
M1-average-payoff=0.5
                             M2-average-payoff=1
```

It is seen that both 111 and 112 have positive average payoffs, and

M2 is the winner.

On the other hand, if an agent Mi has acquired knowledge concerning the payoff function fj of his opponent agent Mj, i.e., he has an accurate alien model, then he can attempt to optimize his performance by selecting appropriate action rules with respect to both payoff functions fi and fj -- i.e. he has both a self-model and an alien model. His action rule (both for sending and responding to messages) is such that the expected value of the difference of his payoff and his opponent's payoff is maximized.

DEFINITION 4: An agent who takes advantage of his knowledge of opponent agent's payoff function and possible action rules, is called a Tricky Dick Agent (TDA).

The state space of a TDA also consists of a single element (fi,fi,fj), where fi is its self-model, and fj is its alien model.

For example, if M1 is HCA, and M2 is TDA, then their respective payoffs can be tabulated as follows:

```
M1-send=0 M2-response=0 M1-payoff= 0 M2-payoff= 1
M2-send=0 M1-response=0 M1-payoff= 0 M2-payoff= 1
M1-average-payoff=0 M2-average-payoff=1
```

If M1 is TDA, and M2 is MCA, then their respective payoffs can be tabulated as follows:

```
M1-send=1
            M2-response=0
                             M1-payoff = -1
                                              112-payoff = -1
                                              M2-payoff = -1
M1-send=1
            M2-response=1
                             M1-payoff=0
                             M1-payoff = -1
                                              M2-payoff = -1
M2-send=0
            MI-response=1
                                              M2-payoff = -1
M2-send=1
            M1-response=1
                             M1-payoff= 0
M1-average-payoff= -0.5
                             M2-average-payoff= -1
```

From the above example, we have the following theorem:

THEOREM 2: A tricky Dick Ni can win over an honest George Nj, even if the honest George Ni loses to the honest George Nj. In other words, it pays to play tricky. To put it in another way, honesty is not the best policy!!

Finally, if both M1 and M2 are tricky Dick, then their respective payoffs can be tabulated as follows:

```
M1-send=1 M2-response=0 M1-payoff= -1 M2-payoff= -1
```

The above example illustrates that when both sides play dirty, they may square off. However, it is also true that they both loses in absolute terms! There is a moral in this example -- live and let live! This is similar to the "prisoner's dilemma" in economics theory.

In a simulation experiment, programs were written to simulate (a) the behavior of HGA, and (b) the behavior of TPA who has perfect knowledge about his opponent's payoff function. All tournament combinations of HGA against HGA, HGA against TDA, and TPA against TDA, using the nineteen different payoff functions, are tried. The results are summarized in Figures Al(a)-(d).

Some general remarks concerning the tournament results can now be stated. From Figure Al, it can be seen that the average payoffs increase according to the payoff functions: some payoff functions (namely, #0 and #1) always yield lowest average payoffs, and some other payoff functions (namely, #17 and #18) always yield highest average payoffs. It is also clear that TDAs generally have advantage over NGAs, because they have information on opponent's payoff functions. However, a more detailed analysis shows the limitations of TDAs.

As an example, we take payoff function #6, with a0=0, a1=0, a2=-1, a3=1, and list its tournament results in Table A2:

tournament	average payoff	prob to win Pl	prob to draw P2	P1+P2
HCA#6 vs HCAs	0.25	0.263	0.368	0.631
HCA#6 vs TDAs	-0.132	0.053	0 • 2 63	0.316
TDA#6 vs HCAs	0.118	0.421	0.316	0.737
TDA#6 vs TDAs	-0.263	0.211	0.316	0.527

Table A2 Tournament results for Agent #6

It can be seen that NGA#6 wins or draws with probability 0.631 vs. HGAs, and that probability decreases to 0.316 vs. TDAs. Its average payoff also decreases from 0.25 to -0.132. On the other hand, TDA#6 wins or draws with probability 0.737 vs. NGAs, and that probability decreases to 0.527 vs. TDAs. Its average payoff also decreases from 0.118 to -0.263. Therefore, we can conclude that if an agent acts as a TDA instead of an NGA, he may win or draw more often, but his average payoff may decrease. The same conclusion can be drawn, by looking at the average (over all payoff functions) of the above values, as shown in Table A3:

tournament	average payoff	prob to win Pl	prob to draw P2	P1+P2
an HGA vs HGAs	0.421	0•338	0.305	0.643
an HGA vs TDAs	0.116	0.211	0.313	0.524
a TDA vs HGAs	0.305	0.476	0.477	0.953
a TDA vs TDAs	0.0	0.338	0.324	0 • 662

Table A3 Average turnament results

Some other remarks are in order: (1) a TDA facing a random opponent is indistinguishable from an HGA with same payoff function; (2) an HGA never wins with negative payoff, although some TDA does; (3) an HGA always draws with itself, a TDA will also draw with itself but may do so with average payoff -1; (4) the HGA with payoff function (0,0,0,0) is the random agent. It never wins, although the TDA with same payoff function may win; (5) if HGA with payoff function fl wins

over TDA with payoff function f2, then the same HGA will also win over HGA with payoff function f2; (6) the only agents (honest or tricky) that never win over any tricky agent, are those with payoff functions #0(1,-1,1,-1), #1(-1,1,-1,1), and #2(0,0,0,0). As a matter of fact, the HGAs with these three payoff functions have similar behavior; (7) An HGA may win over a TDA. For example, HGA with payoff function #17(1,1,-1,-1) can win over any TDA.

A2. Estimation of Alien-Model of HCA

The next problem we wish to investigate is the estimation of payoff functions or estimation of alien-models. A tricky Dick Ms would like to know the payoff function (or alien agent's state) of his opponent Ma. Suppose the opponent's payoff function fa=(a0,a1,a2,a3) is not directly observable. The TDA may try to estimate fa by observing the opponent's message sending/responding behavior.

For a TDA to estimate the payoff function of his opponent (assuming it is known that his opponent is an HGA), he must waste some of his moves to gain information regarding the behavior of his opponent. Specifically, he must examine what his opponent is sending him and what his opponent is responding when he sends messages to his opponent. It is clear that by examining a sequence of messages sent back and forth, the TDA will be able to categorize his opponent into one of several classes; and then for the remainder of the interaction the TDA can react according to the class of the payoff functions rather than to an individual payoff function.

Using Definition 2, the payoff functions of an NGA can be divided

into nine classes. For each class, the external behavior of the NGA is the same.

CLASS A: a0+a1>a2+a3, a0>a2, a1>a3 #17(1,1,-1,-1), #9(1,0,0,-1), #11(0,1,-1,0)

CLASS B: a0+a1>a2+a3, a0>a2, a1=a3 #13(1,0,-1,0)

CLASS C: a0+a1>a2+a3, a0=a2, a1>a3#16(0,1,0,-1)

CLASS D: a0+a1=a2+a3, a0=a2, a1=a3#0(1,-1,1,-1), #1(-1,1,-1,1), #2(0,0,0,0)

CLASS E: a0+a1=a2+a3, a0>a2, a1<a3#3(1,-1,0,0), #7(1,-1,-1,1), #6(0,0,-1,1)

CLASS F: a0+a1=a2+a3, a0<a2, a1>a3 #5(0,0,1,-1), #8(-1,1,1,-1), #4(-1,1,0,0)

CLASS G: a0+a1<a2+a3, a0<a2, a1<a3 #12(0,-1,1,0), #10(-1,0,0,1), #18(-1,-1,1,1)

CLASS II: a0+a1<a2+a3, a0=a2, a1<a3 #15(0,-1,0,1)

CLASS I: a0+a1<a2+a3, a0<a2, a1=a3 #14(-1,0,1,0)

THEOREM 3: Given only the external behavior of an HGA, it is impossible to correctly estimate its payoff function.

Therefore, the TDA can only estimate his opponent's class of payoff functions. However, if the TDA can also observe the current winner, then he can further pinpoint the opponent's payoff function into a smaller class.

In most cases, the categorization of the opponent's class of payoff functions is not sufficient in determining the action rules for a TDA. One way to formulate the action rules for the TDA would be to assume that, given a class of payoff functions, the opponent will always be using the best possible payoff function in that class. In

other words, the TDA will be reacting to the worst case. This TDA will be somewhat limited in its power, and his advantage over an HGA is therefore lessened.

A3. Estimation of Alien-Model of TDA

Similarly, if the opponent is a TDA, its class of payoff functions can also be estimated. Suppose the self TDA is has payoff function (s0,s1,s2,s3), and the alien TDA is has payoff function (a0,a1,a2,a3). The following relations can be deduced:

- (1) If TDA Ma sends 0, then ((a0+a1) (s0+s2)) > ((a2+a3) (s1+s3)), or ((a0-a2) + (a1-a3)) > ((s0-s1) + (s2-s3)).
- (2) If TDA Na sends 1, then ((a2+a3) (s1+s3)) > ((a0+a1) (s0+s2)), or ((a0-a2) + (a1-a3)) < ((s0-s1) + (s2-s3)).
- (3) If Ms sends 0 and TDA Ma responds with 0, then a0-s0 > a2-s1, or a0-a2 > s0-s1.
- (4) If is sends 0 and TDA Ma responds with 1, then a0-a2 < s0-s1.
- (5) If Ns sends 1 and TDA Na responds with 0, then a1-s2 > a3-s3, or a1-a3 > s2-s3.
- (6) If I's sends 1 and TDA I'a responds with 1, then a1-a3 < s2-s3.

From the above relations, the payoff functions of TDA can again be divided into 8 classes:

- CLASS a: a0-a2<s0-s1, a1-a3<s2-s3, ((a0-a2)+(a1-a3))<((s0-s1)+(s2-s3))#0, #1, #2, #8, #10, #12, #14, #15, #18
- CLASS b: a0-a2=s0-s1, a1-a3<s2-s3, ((a0-a2)+(a1-a3))<((s0-s1)+(s2-s3))#3, #6
- CLASS c: a0-a2 < s0-s1, a1-a3 > s2-s3, ((a0-a2)+(a1-a3)) < ((s0-s1)+(s2-s3))
- CLASS d: a0-a2>s0-s1, a1-a3>s2-s3, ((a0-a2)+(a1-a3))>((s0-s1)+(s2-s3))#17
- CLASS e: a0-a2=s0-s1, a1-a3=s2-s3, ((a0-a2)+(a1-a3))=((s0-s1)+(s2-s3))#9, #11

CLASS f: a0-a2 < s0-s1, a1-a3 > s2-s3, ((a0-a2)+(a1-a3))=((s0-s1)+(s2-s3))#16

CLASS g: a0-a2>s0-s1, a1-a3<s2-s3, ((a0-a2)+(a1-a3))=((s0-s1)+(s2-s3))

CLASS h: a0-a2<s0-s1, a1-a3=s2-s3, ((a0-a2)+(a1-a3))<((s0-s1)+(s2-s3))#4, #5

A4. Sophisticated NGA and TDA

In this section, we investigate the use of misinformation in our model. In other words, messages will be encoded by an encoder, before being sent to the opponent. The opponent will use a decoded to decode the incoming messages, before responding to them. We define a sophisticated HGA (SHGA), which sends messages not entirely according to his optimal expected payoff rules, and a sophisticated TDA (STDA), which estimates the payoff function of his opponent SHGA and sends messages according to his estimation of opponent's payoff function class.

SHGA sends messages according to optimal expected payoff rules with probability (1-p), and not according to optimal expected payoff rules with probability p. If $0 \le p < 0.5$, SHGA sends more "right" messages. If p = 0.5, his messages are random. If 0.5 , he sends more "wrong" messages than "right" messages.

STDA observes the action of SHGA for a sufficiently long time. In response to STDA's message 0, if SHGA sends more 0's than 1's, then STDA concludes that a0>a2; if SHGA sends as many 0's as 1's, then STDA concludes that a0=a2; otherwise he concludes that a0 < a2. In this manner, STDA can estimate to which class his opponent's payoff function belongs.

In other words, STDA assumes that the probability p with which

SHGA sends wrong messages is less than or equal to 0.5. Since STDA does not know p, it is natural that he assumes that his opponent will send more right messages than wrong ones.

Therefore, if $0 \le p \le 0.5$, STDA can correctly estimate the class of opponent's payoff function. If p = 0.5, STDA regards his opponent's payoff function to be in Class D. If $0.5 \le p \le 1.0$, STDA cannot correctly estimate the class. For example, functions belonging to Class A are regarded to be in Class G, and the conditions are interpreted to be their opposites. Similarly, classes E and F, B and I, C and H, are naturally confused.

Table A4 illustrates the average probabilities of SHGA and STDA to win or draw over all possible cases, for some fixed values of p. (The results are averaged over all payoff functions in a function class estimated by STDA.) The results are also depicted in Figure A2. Some general conclusions can now be drawn.

	Probability T	o Win Or Draw
р	SHGA	STDA
0.0	0.682	0.679
0.2	0.448	0.684
0.4	0.344	0.788
0.49	0.323	0.809
0.5	0.540	0.810
0.51	0.417	0.711
0.6	0.404	0.724
0.8	0.354	0.775
1.0	0.347	0.787

Table A4 Probabilities of SHGA and STDA to win or draw

Mhen p is 0, STDA knows only the class to which SECA's payoff function belongs, and his strategy of maximizing the expected value of difference of their respective payoffs does not always succeed. Instead, SHGA always sends messages maximizing his expected payoff.

So in this case, SHGA (who wins or draws with probability 0.682) is actually stronger than STDA (who wins or draws with probability 0.679).

As p increases toward 0.5, STDA gets stronger, and SHGA gets weaker, because the situation concerning STDA remains the same, but SHGA no longer always sends messages maximizing his expected payoff. In particular, when p = 0.49, STDA wins or draws with probability 0.809, and that for SHGA becomes 0.323.

When p reaches 0.5, SHGA becomes the random opponent. Beyond this point, SHGA sends more wrong messages than right messages. But as the STDA assumes his opponent's payoff function to be in the opposite class, his strategy to maximize the difference of their payoffs also hardly succeeds. No general conclusion can be drawn, except that STDA is stronger than SHGA when p is larger than 0.5. It should be noted, however, that SHGA is stronger when he always sends wrong messages (at p=1.0, he wins or draws with probability 0.347), than when he sometimes sends wrong messages (at p=0.4, he wins or draws with probability 0.344).

A5. N-Agent Message Exchange

We can now define n-agent competitive information exchange. Suppose there are agents MI, M2, ..., Mn. Each agent MI will take turns in selecting another agent MJ, and send him a message, who must then respond. In other words, agents send (and respond to) messages in round-robin fashion. Some agents may want to become cooperating agents, because they have complementary payoff functions. For example, if MI likes to receive more 0's, and M2 likes to receive more

I's, then they can cooperate, because each stands to pain. Therefore, the action rules of an agent need to include rules to select (over a period of interactions) those agents that are cooperating with him.

A computer program was written to simulate n-agent information exchange [CHANG79]. The program simulates 19 MGAs communicating with one another, each agent having a different payoff function. Two rules are used in round-robin message exchange. Fulle 1: An agent will select those opponents who will give him positive average payoffs to communicate with. Dule 2: An agent will decline to communicate with those opponents who will give him negative average payoffs.

In the beginning of the simulation experiment, each agent will communicate with all other agents. After applying bule 1, it is noted that average payoffs of most agents do show an increase. After the application of bule 2, in addition to increase of payoffs, it is noted that cliques have been formed. Unless payoff functions are changed, a definite ressage exchange pattern among the verious participating agents have been formed. Therefore, if we regard the ressage exchange pattern as a graph, we can define the <u>information distance</u> between two information nodes in an information system as the shortest path from one information node to another information node, where the are weight between two adjacent information nodes is the information distance between these two agents, as defined in Section 7. If no paths exist, then the information distance between two information nodes is infinity.

The information distance matrix A for the nineteen NGAs is illustrated below, where an asterisk indicates an infinite information distance.

```
2
         1
           2
               2
                 1
                      2
                        2
                            2
                                3
             1
                    1
                          1
                              1
               2
         2
           2
             2
                      2 2
                            2
     2
       1
                 1 1
                          1
                              1
       2 1
           2
             1
               2
                 2
                   2
                      1
                          2
                            2
   2
     1
                        1
                              1
                                2
                                  1
   1 2 2 2 1 2 2 2 1 2 1
                          2 1
                              2
                                1
                                  3 1
 * 2 1 2 1 2 1 2 2 2 1 2 2 2 1
                                3 1 1 1
         2
       2
           2 2 2
                 1 1
                        2
                            2
       1 2 2 2 1
                 2 2 1
                 2
                          2 1
                              2
         2
           1 2
                   1
                      3 I
                2
                    3
           1
             2
               2
                 2
                   1
                      2
         1
                        1
                              2
   2 1 1 2 2 2 1
                 2
                   2 1
                        2
                          1
                              2
                                2
 * 1
     2 2 2 1 2 2 1 1
                      2
                        1
                          1
 * 2 1 1 1 2 1 1 2 2 1 2 2
     3 1 2 1 3 3 2 2 3 1 2 1 2 2 3 2 2
 * 3 2 2 1 3 1 2 3 3 2
                        2 1 2 1
                                3 2 2 2
 * 2 1 2 1 1 1 1 1 1 2 1 2 1 1 2 2 1 2
* 1 2 1 2 1 1 1 1 2 1 2 1 1 1 1 2 2 2 1
```

It is noted that HGA #0(1,-1,1,-1) does not communicate with any other agent. If we define a k-clique as a set of nodes in which the information distance between any two nodes is no greater than k, then HGAs #1, #3, #18 form a 1-clique, HGAs #2, #10, #16 form a 2-clique, and all the HGAs, excluding #0, form a 3-clique, etc. Disregarding HGA #0, the average information distance is 1.617. The average information distance for HGA #16 to other HGAs excluding #0 is 2.055, which is the maximum; and for HGA #13 is 1.388, which is the minimum.

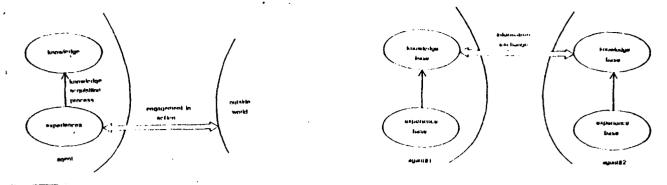


Figure 1

Figure 2

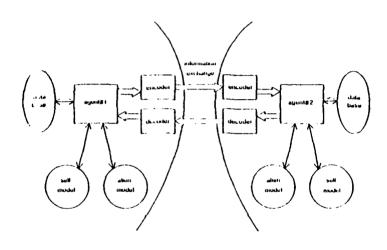


Figure 3

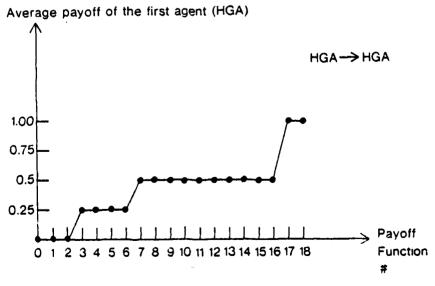


Figure Al(a)

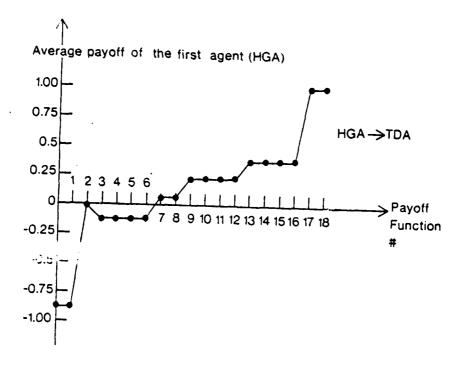


Figure A1(b)

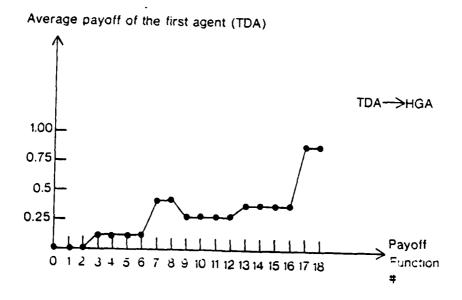


Figure Al(c)

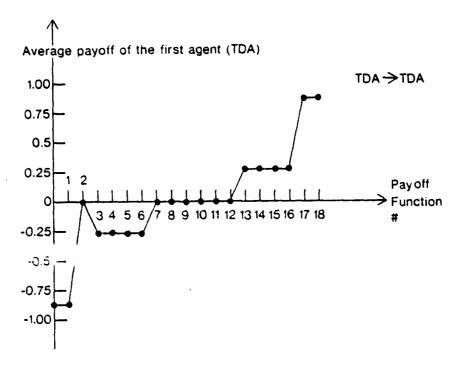


Figure Al(d)

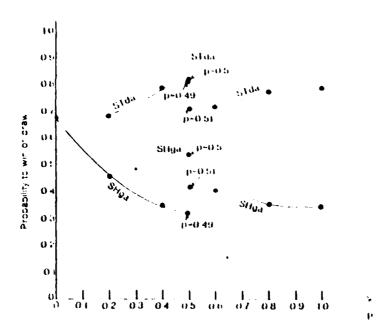


Figure A2

